OCR
RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS

4723 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Solve the inequality $|2x - 5| > |x + 1|$. **[5]**

2 It is given that $p = e^{280}$ and $q = e^{300}$.

(i) Use logarithm properties to show that $\ln\left(\frac{ep^2}{q}\right) = 261$. **[3]**

(ii) Find the smallest integer n which satisfies the inequality $5^n > pq$. **[3]**

3 It is given that θ is the acute angle such that $\sec\theta \sin\theta = 36\cot\theta$.

(i) Show that $\tan\theta = 6$. **[3]**

(ii) Hence, using an appropriate formula in each case, find the exact value of

(a) $\tan(\theta - 45°)$, **[2]**

(b) $\tan 2\theta$. **[2]**

4 **(a)** Show that $\int_0^4 \frac{18}{\sqrt{6x+1}}\,dx = 24$. **[4]**

(b) Find $\int_0^1 (e^x + 2)^2\,dx$, giving your answer in terms of e. **[4]**

5 **(i)** It is given that k is a positive constant. By sketching the graphs of

$$y = 14 - x^2 \text{ and } y = k\ln x$$

on a single diagram, show that the equation

$$14 - x^2 = k\ln x$$

has exactly one real root. **[3]**

(ii) The real root of the equation $14 - x^2 = 3\ln x$ is denoted by α.

(a) Find by calculation the pair of consecutive integers between which α lies. **[3]**

(b) Use the iterative formula $x_{n+1} = \sqrt{14 - 3\ln x_n}$, with a suitable starting value, to find α. Show the result of each iteration, and give α correct to 2 decimal places. **[4]**

6 The volume, $V\,\text{m}^3$, of liquid in a container is given by

$$V = (3h^2 + 4)^{\frac{3}{2}} - 8\,,$$

where h m is the depth of the liquid.

(i) Find the value of $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.6$, giving your answer correct to 2 decimal places. **[4]**

(ii) Liquid is leaking from the container. It is observed that, when the depth of the liquid is 0.6 m, the depth is decreasing at a rate of 0.015 m per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is 0.6 m. **[3]**

7 The function f is defined for all real values of x by $\text{f}(x) = 2x + 5$. The function g is defined for all real values of x and is such that $\text{g}^{-1}(x) = \sqrt[3]{x - a}$, where a is a constant. It is given that $\text{fg}^{-1}(12) = 9$. Find the value of a and hence solve the equation $\text{gf}(x) = 68$. **[7]**

8 **(i)** Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

(a) solve the equation $3\sin\theta + 4\cos\theta + 1 = 0$, giving all solutions for which $-180° < \theta < 180°$, **[4]**

(b) find the values of the positive constants k and c such that

$$-37 \leqslant k(3\sin\theta + 4\cos\theta) + c \leqslant 43$$

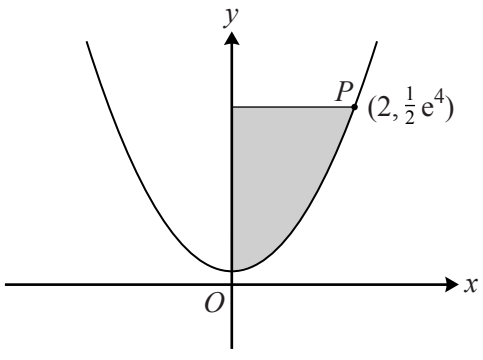
for all values of θ. **[4]**

9 **(i)** Show that the derivative with respect to y of

$$y\ln(2y) - y$$

is $\ln(2y)$. **[3]**

(ii)



The diagram shows the curve with equation $y = \frac{1}{2}\text{e}^{x^2}$. The point $P(2, \frac{1}{2}\text{e}^4)$ lies on the curve. The shaded region is bounded by the curve and the lines $x = 0$ and $y = \frac{1}{2}\text{e}^4$. Find the exact volume of the solid produced when the shaded region is rotated completely about the y-axis. **[6]**

(iii) Hence find the volume of the solid produced when the region bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$ is rotated completely about the y-axis. **[2]**

Question			Answer	Marks	Guidance	
1			Attempt process for finding critical values	M1	squaring both sides, 2 linear eqns, ineqs, …	If using quadratic, need to go as far as factorising or substituting in formula for M1; if using two linear eqns or ineqs, signs of $2x$ and x must be same in one, different in the other for M1
			Obtain $\frac{4}{3}$	A1		
			Obtain 6	A1		
			Attempt process for inequality involving two critical values	M1	sketch, table, …; implied by plausible soln	
			Obtain $x < \frac{4}{3}$, $x > 6$	A1	A0 for use of $\leq$ and/or $\geq$	
				[5]		
2	(i)		EITHER			
			Attempt use of at least one logarithm property correctly applied to $\ln(\frac{ep^2}{q})$	M1	not including $\ln e = 1$; such as $... = \ln ep^2 - \ln q$ for example	
			Obtain 261 legitimately with necessary detail seen	A2	AG; award A1 if nothing wrong but not quite enough detail or if there is one slip on way to 261	
				[3]		
			OR			
			Express $\frac{ep^2}{q}$ in form e^n	M1	with correct treatment of powers	
			Obtain e^{261} and hence 261	A2	AG; award A1 if nothing wrong but not quite enough detail to be fully convincing	
2	(ii)		Introduce logarithms and bring power down	M1	relating $n \ln 5$ to a constant; if using base 5 or base 10, no powers must remain on right-hand side	
			Obtain $n \ln 5 > 580$	A1	or equiv (such as $n > 580 \log_5 e$ or $n \log 5 > 580 \log e$); allow eqn at this stage	
			State single integer 361	A1	not $n > 360$ nor $n \geq 361$	
				[3]		

Question			Answer	Marks	Guidance	
3	(i)		Use $\sec\theta = \dfrac{1}{\cos\theta}$ Attempt to express in terms of $\tan\theta$ only Obtain $\tan^2\theta = 36$ and hence $\tan\theta = 6$	B1 M1 A1 **[3]**	AG; necessary detail needed (but no need to justify exclusion of $\tan\theta = -6$)	
3	(ii)	(a)	Substitute 6 in attempt at formula Obtain $\frac{5}{7}$	M1 A1 **[2]**	of form $\dfrac{\tan\theta \pm \tan 45°}{1 \mp \tan\theta \tan 45°}$ with different signs in numerator and denominator or exact equiv	any apparent use of angle 80.5.. means M0 answer only: 0/2
3	(ii)	(b)	Substitute 6 in attempt at formula Obtain $-\frac{12}{35}$	M1 A1 **[2]**	of form $\dfrac{\tan\theta + \tan\theta}{1 \pm \tan\theta \tan\theta}$ or exact equiv; allow $\frac{12}{-35}$	any apparent use of angle 80.5.. means M0 answer only: 0/2
4	(a)		Obtain integral of form $k(6x+1)^{\frac{1}{2}}$ Obtain $6(6x+1)^{\frac{1}{2}}$ Substitute both limits and subtract Obtain 30 – 6 and hence 24	*M1 A1 M1 A1 **[4]**	any constant k or (unsimplified) equiv dep *M AG; necessary detail needed	
4	(b)		Attempt expansion of integrand Integrate e^{kx} to obtain $\frac{1}{k}e^{kx}$ Obtain $\frac{1}{2}e^{2x} + 4e^{x} + 4x$ Obtain $\frac{1}{2}e^{2} + 4e - \frac{1}{2}$	M1 M1 A1 A1 **[4]**	to obtain (at least) 3 terms for any constant k other than 1 allow $+c$ at this stage or equiv in terms of e simplified to three terms; no $+c$ now	

Question			Answer	Marks	Guidance	
5	(i)		Sketch (more or less) correct $y = 14 - x^2$ Sketch (more or less) correct $y = k \ln x$ Indicate one root ('blob' on sketch or written reference to one intersection or …)	B1 B1 B1 **[3]**	assessed separately from other graph; must exist in all four quadrants; ignore any intercepts given assessed separately from other graph; must exist in first and fourth quadrants; if clearly meets y-axis award B0; if clear maximum point in first quadrant award B0 dependent on both curves being correct in first quadrant and there being no possibility, from their graphs, of further points of intersection elsewhere	
5	(ii)	(a)	Calculate values for at least 2 integers Obtain correct values for $x = 3$ and $x = 4$ State 3 and 4	M1 A1 A1 **[3]**	 $14 - x^2 - 3\ln x$: 1.7 –6.2 $14 - x^2$, $3\ln x$: 5, 3.3 –2, 4.2 following correct calculations	
5	(ii)	(b)	Obtain correct first iterate Attempt iteration process Obtain at least 3 correct iterates in all Obtain 3.24	B1 M1 A1 A1 **[4]**	having started with any positive value; B1 available if 'iteration' never goes beyond a first iterate; implied by plausible sequence of values showing at least 2 d.p. answer required to exactly 2 d.p; not given for 3.24 as the final iterate in a sequence, i.e. needs an indication (perhaps just underlining) that value of α found [3 → 3.27172 → 3.23173 → 3.23743 → 3.23661 3.5 → 3.20027 → 3.24196 → 3.23596 → 3.23682 4 → 3.13706 → 3.25118 → 3.23465 → 3.23701]	

Question			Answer	Marks	Guidance	
6	(i)		Attempt use of chain rule	*M1	to obtain derivative of form $kh(3h^2+4)^n$, any non-zero constants k and n condone retention of -8	
			Obtain $9h(3h^2+4)^{\frac{1}{2}}$	A1	or (unsimplified) equiv; no -8 here	
			Substitute 0.6 in attempt at first derivative	M1	dep *M; condone retention of -8 here; implied by their value following wrong derivative if no working seen	
			Obtain 12.17	A1 **[4]**	or greater accuracy	
6	(ii)		State or imply that $\frac{dh}{dt} = -0.015$ or 0.015	B1	implied by use in calculation with part (i) answer	
			Carry out multiplication of $(\pm)0.015$ and answer from part (i)	M1		
			Obtain 0.18 or -0.18 (whatever this value is claimed to be)	A1 **[3]**	or greater accuracy; condone absence or misuse of negative signs throughout; ignore units; allow for answer rounding to 0.18 following slight inaccuracy due to use of 12.18 or 12.2 or …	
7			Show composition of functions	M1	the right way round; or equiv	
			Obtain $2\sqrt[3]{12-a}+5=9$	A1	or equiv	
			Obtain $a=4$	A1		
			EITHER			
			Attempt to find $g(x)$	*M1	obtaining px^3+q or py^3+q form	
			Obtain $(2x+5)^3+4=68$	A1ft	following their value of a	
			Attempt solution of equation	M1	dep *M; earned at stage $2x+5=...$; if expanding to produce cubic equation, earned with attempt at linear and quadratic factors	
			Obtain $-\frac{1}{2}$	A1 **[7]**	and no others; dependent on correct work throughout	
			OR			
			State or imply $f(x)=g^{-1}(68)$	B2		
			Attempt solution of equation of form $2x+5=\sqrt[3]{68-4}$	M1		
			Obtain $-\frac{1}{2}$	A1		

Question			Answer	Marks	Guidance	
8	(i)		State $R = 5$ Attempt to find value of α Obtain 53.1	B1 M1 A1 **[3]**	 implied by correct value or its complement allow $\tan^{-1}\frac{4}{3}$	
8	(ii)	(a)	Attempt to find at least one value of $\theta + \alpha$ Obtain 1 correct value of θ (–64.7 or 138) Attempt correct process to find the second value Obtain second value of θ (138 or –64.7)	M1 A1 M1 A1 **[4]**	(should be -168.5 or -11.5 or 191.5 or …) allow ± 0.1 in answer and greater accuracy involving a positive value of $\sin^{-1}(-\frac{1}{5})$ and subtraction of their α allow ± 0.1 in answer and greater accuracy; and no others between –180 and 180	note that 138 needs to be obtained legitimately from positive value of $\sin^{-1}(-\frac{1}{5})$ and not from $180 - 41.6$ answers only: 0/4
8	(ii)	(b)	Use –1 as minimum or 1 as maximum value of $\sin(\theta + \alpha)$ Relate $-5k + c$ to -37 and $5k + c$ to 43 Attempt solution of pair of linear eqns Obtain $k = 8$ and $c = 3$	*M1 A1 M1 A1 **[4]**	 as equations or inequalities dep *M; must be equations now **SC**: both $k = 8$ and $c = 3$ obtained with no working or from unconvincing working, award B2 (i.e. max 2/4)	 Note that alternative solutions may occur. If mathematically sound, all 4 marks are available; if work is not fully convincing, apply SC

Question			Answer	Marks	Guidance	
9	(i)		Attempt use of product rule to produce the form $\ln 2y + y \times \frac{a}{by}$	M1		Note that product rule may be applied to expression in form $y(\ln 2y - 1)$
			Obtain correct $\ln 2y + y \times \frac{2}{2y}$ ...	A1	or equiv	
			Obtain complete $\ln 2y + 1 - 1$ and confirm	A1 **[3]**	AG; necessary detail needed	
9	(ii)		Attempt to rearrange eqn to $x =$ …or $x^2 = ...$	M1	obtaining form $p \ln qy$	
			Obtain $x = \sqrt{\ln 2y}$ or $x^2 = \ln 2y$	A1		
			State or imply volume is $\int \pi \ln 2y \, dy$	A1ft	following their $x =$ … or $x^2 = ...$; condone absence of dy; condone presence of dx; no need for limits here; π may be implied by its first appearance later in solution	
			Integrate using result of part (i)	M1		
			Attempt to use limits $\frac{1}{2}$ and $\frac{1}{2}e^4$ correctly with expression involving y	M1		
			Obtain $\frac{1}{2}\pi(3e^4 + 1)$	A1 **[6]**	or equiv involving two terms; dependent on correct work throughout part (ii)	
9	(iii)		Subtract answer to part (ii) from $2\pi e^4$ …	M1	… or its decimal equivalent	
			Obtain $\frac{1}{2}\pi(e^4 - 1)$	A1 **[2]**	or exact equiv involving two terms	